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MUON SHIELDING:

MULTIPLE COULOMB SCATTERING OF MUONS WITH ENERGY LOSS

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About 1940, Fermi found the distribution function for the lateral and angular displacements of charged particles which undergo multiple elastic scattering in passing through a layer of matter. In his treatment, the energy loss which the particles suffer due to ionizing collisions is neglected. Rossi and Greisen[1] worked the theory out in detail and Rossi's book[2] discusses it. One result which is commonly used is the prediction of the mean scattering angle:

$$\langle\Theta^2\rangle_{av} = \left(\frac{E_s}{\beta cp}\right)^2 \frac{x}{x_o} \qquad (1a)$$

or in more standard rotation

$$\langle\Theta\rangle_{rms} = \frac{15 \text{ (MeV)}}{PV \text{ (MeV)}}\sqrt{\frac{x}{x_o}} \qquad (1b)$$

where x_o = radiation length.

The form of the angular scattering was Gaussian with this being the "half-width". Moliere[3] improved the theory by introducing a different angular distribution, but for small angle scattering the Gaussian is still a good approximation.

Eyges[4] took the theory and modified it to include the energy loss for ionizing collisions. In Rossi and Greisen's notation the diffusion equation for the distribution function $F(t, y, \Theta)$ is:

$$\frac{\partial F}{\partial t} = -\Theta \frac{\partial F}{\partial y} + \frac{1}{W^2} \frac{\partial^2 F}{\partial \Theta^2} \tag{2}$$

where t is the thickness traversed in units of radiation lengths, y is the lateral drift, Θ is the angular scattering, and $W = 2p\beta/E_s$. He assumed that p and β are functions of t - this is where the energy loss dE/dx is used. W is assumed to be a known function of t, although not necessarily one for which there is an analytic expression. Neglected is the fact that a particle at t has traveled a somewhat greater distance than t due to the deviations caused by scattering. For multiple scattering of high energy particles these deviations are small and the approximation will be a good one.

The solution is:

$$F(t, y, \Theta) = \frac{1}{2\pi (B(t))^2} \exp\left(\frac{-\Theta^2 A_2 - 2y\Theta A_1 + y^2 A_0}{4A_0 B}\right) \tag{3}$$

where

$$B(t) = A_0 A_2 - A_1{}^2, \tag{4a}$$

$$A_0(t) = \int_0^t \frac{dn}{W^2(n)} \tag{4b}$$

$$A_1(t) = \int_0^t \frac{(t-n)\,dn}{W^2(n)} \tag{4c}$$

$$A_2(t) = \int_0^t \frac{(t-n)^2\,dn}{W^2(n)} \tag{4d}$$

where W has the same definition as earlier and n is a dummy variable of thickness. If W^2 is a constant, this solution reduces to the Fermi solution as given by Rossi and Greisen.

If you integrate over y, you get the angular distribution irrespective of displacement:

$$\int_{-\infty}^{\infty} F(t,\ y,\ \Theta)\,dy = \frac{1}{2(\pi A_0)^{1/2}} \exp\left(-\frac{\Theta^2}{4A_0}\right). \tag{5}$$

Similarily, the lateral distribution independent of angle is

$$\int_{-\infty}^{\infty} F(t,\ y,\ \Theta)\,d\Theta = \frac{1}{2(\pi A_2)^{1/2}} \exp\left(-\frac{y^2}{4A_2}\right). \tag{6}$$

Some insight can be gained into formulas 5 and 6 by comparing them. The angular distribution (5) depends on t through A_0, i.e., through $\int_0^t dn/W^2(n)$ whereas the lateral distribution (6) depends on t through A_2 or $\int_0^t \left[(t-n)^2/W^2(n)\right]dn$. Because of the factor $(t-n)^2$ large values of (t-n) are weighted more heavily in the last integral than in the preceding one. This is caused by the fact that a given angular scattering produces a larger lateral drift at t the farther from t that it occurs, whereas all angular deflections at any intermediate thickness contribute equally to the total angular scattering at t.

In general, the integrals A_0, A_1, and A_2 which determine these distributions can be obtained by simple numerical integration using the range versus momentum tables which have already been published for the case of muons[5]. For a few special cases the A's can be found by direct integration. For example, for a high energy particle one can assume a constant dE/dx. Then, the momentum p of a particle at depth t radiation lengths is $p_0-\varepsilon t$ where ε is the constant momentum loss per radiation length. Then $W^2(t) = \left[4(p_0-\varepsilon t)^2/E_s^{\,2}\right]$, hence, the A's become:

$$A_0\ (t) = \frac{E_s^{\ 2}}{4p_0\,(p_0-\varepsilon t)}\,t \tag{7a}$$

$$A_1\ (t) = \frac{E_s^{\ 2}}{4\varepsilon^2}\left[\ln\frac{p_0}{p_0-\varepsilon t} - \frac{\varepsilon t}{p_0}\right] \tag{7b}$$

$$A_2\ (t) = \frac{E_s^{\ 2}}{4\varepsilon^2}\left[2t - \frac{t^2\varepsilon}{p_0} - 2\,\frac{(p_0-\varepsilon t)}{\varepsilon}\,\ln\frac{p_0}{p_0-\varepsilon t}\right] \tag{7c}$$

A direct comparison can now be made with the Fermi theory of multiple scattering. Under these approximations (i.e., $\beta=1$), Eq. 1a becomes

$$\langle\Theta^2\rangle_{av} = \frac{E_s^{\ 2}}{p_0^{\ 2}}\,t \tag{8}$$

The Gaussian half-width of the angular distribution function in the Eyges theory is

$$\langle\Theta^2\rangle_{av} = 4A_0 = \frac{E_s^{\ 2}}{p_0\,(p_0-\varepsilon t)}\,t \tag{9}$$

If the constant momentum loss $\varepsilon = 0$, the two theories have identical angular distributions. Such a trivial comparison is not possible between the lateral distributions.

In designing muon shields one is interested in the lateral displacement function, Eq. 6. For cylindrical geometry this equation becomes

$$I\ (t,\ r) = \frac{1}{4\pi A_2}\,\exp -\left(\frac{r^2}{4A_2}\right) \tag{10}$$

where the A_2 is the same function of t as given in Eq. 7c, and r is the displacement from the original direction. Equation 10 can be interpreted as the solution for a case of a narrow beam of particles of a given incident energy normally incident on a semi-infinite slab of material. I is the current density in a plane at t normal to the direction of the incident particle. By calculating this function at various depths and many radii, one can plot the diffusion of a particle of given incident energy in a block of material. Shown in figure 1 are the isocurrent density lines for muons of various energies in iron. The choice of 10^{-7} is arbitrary, but has the meaning that for 1 muon/cm^2 incident on an infinite block of iron, the current density along this line is 10^{-7} muons/cm^2. For the 200 GeV case other isocurrent density curves are also shown. The predominate shape of the isocurrent density lines is that of a slightly flared horn. A similar set of curves for soil is shown in figure 2. A muon shield is simply made up of a superposition of many of these horns chosen correctly and added in accordance with the incident muon flux as a function of energy and angle.

In designing the muon shield for the production target in an experimental area this superposition is a rather complicated process which take account of the muon flux as a function of energy and angle for a given production model. Results on muon shields for such an area have been reported in a previous TM[6].

The design of a muon shield at the end of an experimental secondary beam line is much simpler. Here the angular dependence of the muons has been suppressed since all muons emerge more or less parallel from the end of the beam line. Then, knowing the muon flux as a function of energy the appropriate horns could be superimposed to provide shielding. A crude example of how this would work can be shown with the aid of figure 2. Suppose that emerging from a given beam line are the following fluxes:

$E\mu$ (GeV)	Flux (Muons/sec/proton)
0 - 60	10^{-6}
60 - 100	10^{-6}
100 - 140	10^{-6}
140 - 180	10^{-6}

The desired flux at the surface of a muon shield is 10^{-13} muons/cm^2/sec/proton. Therefore, the necessary attenuation is 10^{-7} The shield could be crudely designed by superimposing the 10^{-7} muon horns for the energies of 40, 80, 120, and 160 GeV and taking the envelope out of these horns as the required shield shape. In actual practice the energy bins would be taken much smaller and the shield would have a smooth outer edge.

Similarly, the design of a lateral muon shield for a bending magnet can be envisioned as superimposing the various horns fanned out to account for the muon fluxes and the angles at which each muon momentum emerges from the magnet.

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Figure 1

Muon Horns for Iron
Isocurrent Density Curves
Iron Density: 7.87 $gm cm^{-3}$

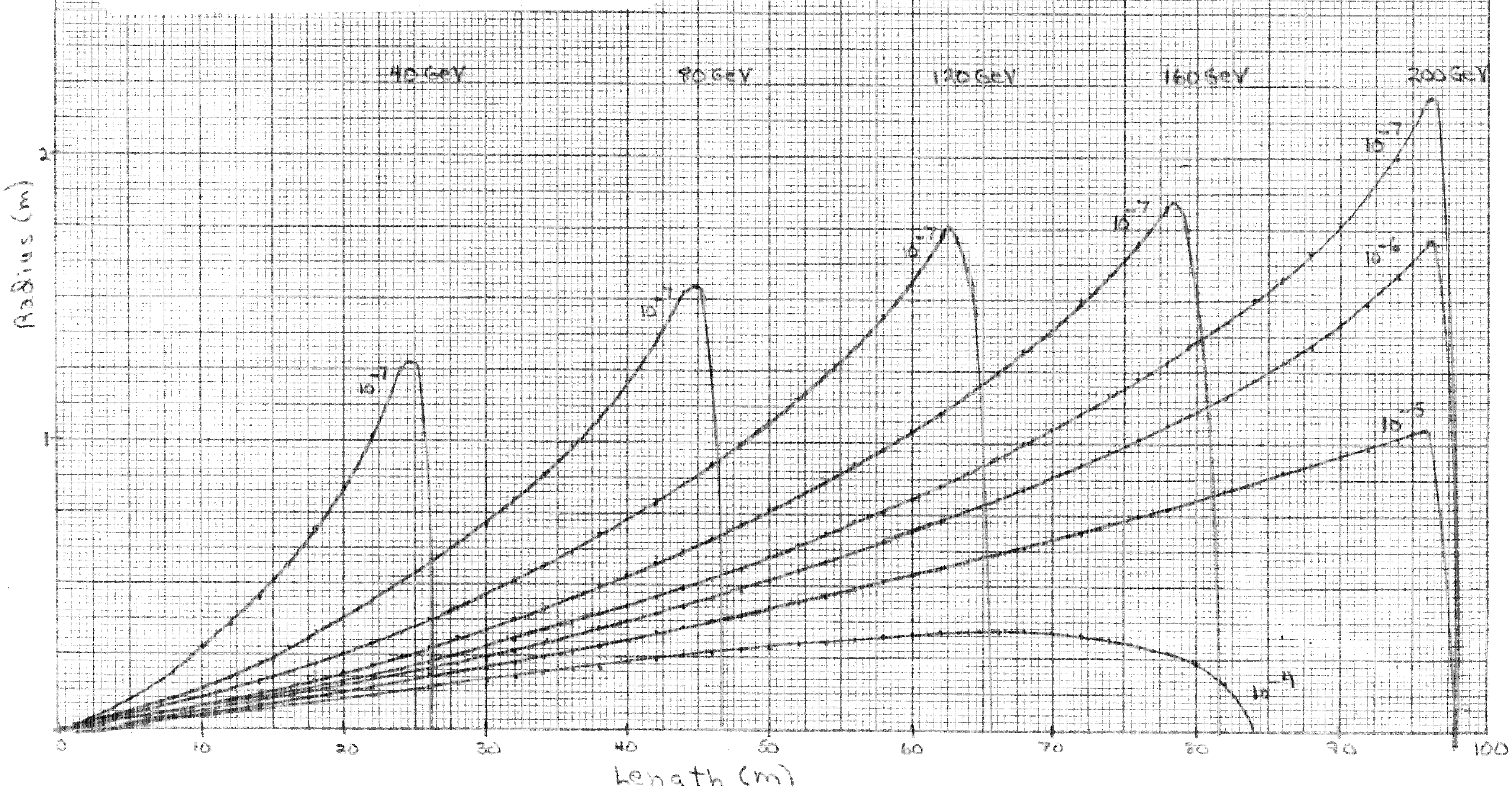

Figure 2.

Muon Horns for Soil
Isocurrent Density Curves
Soil Density: 2.0 $gmcm^{-3}$